# INTERDISCIPLINARY ACTIVITIES TO IMPROVE THE LEARNING METHODOLOGY PERFORMED IN MECHANICAL ENGINEERING DEGREE STUDIES

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## Abstract

The multidisciplinary character of engineering studies should be considered, in the process of teaching and learning mathematics, as a key to ensure that students incorporate knowledge in a meaningful way so as to be used in the development of basic and applied technologies.

The current work describes methodological changes implemented in Advanced Computing at the Mechanical Engineering studies, which we believe to be innovative, aimed to integrate disciplines, with an approach based on the complexity of fluid flow systems and mathematical models, so as to introduce small working groups of students in research work and to pursuit new knowledge.

The so-called virtual labs have emerged in the area of engineering education as a potential alternative to the traditional laboratories. This new learning system is possible due to the capability offered by the recent technological advancements that allow us to resolve symbolic and also numerically each system, which can be represented by a mathematical model and designed using simulation models, these present outstanding advantages in the process of teaching in several disciplines.

This change in the context of learning in Engineering aims to place emphasis on the interpretation of the various parameters of the systems under study, using tools of symbolic manipulation; and the generation of a field of collaborative work; allowing the formation of basic capabilities; having an impact in the student's multidisciplinary formation.

This presentation includes the planning, analysis and selection of contents in the study of models related with Fluids Mechanics, by employing a platform of multiphysics simulation where Mathematics and varied Physics systems can promote and facilitate the conceptualization of complex models.

Keywords: multidisciplinary, virtual labs, fluid flow, computational mathematics.

## 1 INTRODUCTION

In an age characterized by new dimensions of complexity, scale and uncertainty, many challenges require solutions that are beyond the reach of one thought discipline. More and more frequently, the advances in science and engineering that will have the greatest impact are those born at the frontiers of more than one engineering discipline. The benefits of multidisciplinary thinking - and the shortcomings of a world that has been "understood" primarily by specialization - have been apparent for several decades.

While the concept of multidisciplinary thinking, or "multidisciplinarity," is not new, it has in recent years emerged as a pervasive term, gaining popularity both in science and in policy contexts. Multidisciplinarity traces its roots to the second half of the 20$^{th}$ Century, with the cross-fertilization among the sub-branches of physics, the development of grand simplifying concepts, the emergence of systems theory and of new fields such as biochemistry, radio astronomy and plate tectonics.

Multidisciplinary engineering refers to engineering that engages one or more areas of engineering (e.g. mechanical, chemical electrical, biomedical, etc.), as well as other sciences or technical disciplines. Multidisciplinary engineering often requires team work. For instance, a mechanical engineer works with a biologist to design a heart valve. In this example, the teammates work together, each contributing their own expertise to solving the problem. Multidisciplinarity additionally refers to the development of conceptual links using a perspective in one discipline to modify a perspective in another discipline, or using research techniques developed in one discipline to elaborate a theoretical framework in another.


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In search of a multidisciplinary training the incorporation of technology in higher education allow significant changes in the teaching and learning process that impact on engineering careers, especially in the subjects of Mathematics area. The insertion of specific software and computational tools which are increasingly powerful, allow making the modifications needed to achieve this purpose.

With the objective to carry out these changes we need to align the University curricula not only to the new work methods that allow intellectual development stimulation but also have a tendency to a multidisciplinary approach. The goal is not only to teach and learn only mathematics, is also doing it by facing with the stimulation of real cases and simple systems that lead the student to have a look at real situations.

The development of numerical methods and the advent of simulation platforms consider the need to guide the methodological approach of Higher Mathematics studies towards a new kind of teaching that modify the learning sequence. Driving the educational system approach to multidisciplinary models has an impact to the student's cognitive process.

Multiphysics stimulation software create an engineering environment where learning opportunities and motivation increases exponentially. Moreover the comprehension improvement and ease of learning complex subjects are improved when trying to learn fluid flow processes with educational purposes, processes that students could not observe and reason in the absence of these resources.

This paper recounts the experiences in Advanced Calculus class when analyzing the fluid flow from two different perspectives, the first aims to work by using mathematics software applying the subject of analytic functions of complex variables, and see the conclusions arrived, the second is to perform the same job but with a multiphysics simulation platform.

Currently, to run Applied Mathematics contents in Engineering studies in a significantly manner, it is convenient to enrich the teaching and learning process by implementing thematic oriented to present models that integrate different disciplines. But not only choosing these models is important, but also the means chosen for its resolution.

It is noteworthy that students are encouraged when they leave aside traditional forms of paradigm-type problem solving, and when they are imposed to a significant and reflexive learning dynamic that involves engineering situations which ignore ideality and are related to the ones to be carried out in practice. Sometimes it is observed that the students do not follow abstraction and generalization processes, or find it difficult to adapt to them. This is why the trend in contemporary education needs the implementation of a system that places the student in centred processes that identify him as an active and reflective learning subject.

To carry out these changes that aim at leading the acquisition of new ways of thinking and reasoning, it is necessary for the teacher to reformulate the methods implemented in the class and is adapted to work in other areas where available modelling and simulation are applied [5].

## 2 APPLIED METHODOLOGIES

This engineering analysis work is based on a design and planning computer aided system. It is important to create a learning space at the higher education system where developed a set of activities and communicational expressions as the fundamental line of the educational process. It will organize theoretical and practical activities where students perform technological applications with the topics developed in class, linking the themes of the subject, in different subjects of the same or different levels of the area, as well as with other disciplines through solving projects whose complexity is conditioned only by the basic knowledge that students have. Teaching strategies are established based on different practical activities without neglecting the theoretical foundation, with a multidisciplinary approach.

In the areas of professional cycle and following a multidisciplinary line, training activities are proposed, with a strong focus on professional activity carried out, showing industrial applications, designing projects with clear goals and objectives that allow the continuous update and coordination of activities in the areas of knowledge of the university studies. It is essential that students, from initiation to completion of their studies in their chosen field make use of computational methods. This will strengthen the student's unified vision between mathematics and its applications and will give the essential tools for their professional work.

"When students learn in the same way they will act as Engineers, in their professional development, they will acquire a real meaningful learning." Activities involved in this project include training teachers to implement the project in Basic Sciences in Chemical, Mechanical, Civil and Electrical majors, conducting interdisciplinary workshops with the use of specific software and tailor-made training materials, training scholars in teaching and researches activities, developing teaching materials in electronic format to implement whether dual or distance education.

It is also intended to support planning and implementation of curricular activities to develop dual and distance learning through the institutional web site. This involves training teachers to develop activities suitable for distance learning and the ability to access materials and resources with the state of the art technology and bibliographical material as well as scientific publications throughout the working team[10].

# 3 TEACHING STRATEGIES

To improve their learning experience students need to have sufficient prior knowledge from which to address the content proposed, in order to establish more complex and rich relations. Therefore, it is initially convenient to help the student to remember, rearrange or assimilate their prior background related with the new content, in order to successfully address the learning program, designing cognitive bridges between the new content and structure of knowledge that the student has - advance organizers - for that purpose appropriate strategies are developed to place students in a favorable position to learn. This involves an intense activity by the student and a real commitment of teachers in regard to the directionality, coordination and learning support.

In this regard, an integrated learning is developed - theoretical and practical and theoretical technology - in an attempt to differentiate experience based on: dialogue, convergence criteria and active student participation. We must ensure that students leave their passive role, acquiring memory ability that prevents think for themselves and create. The right and properly sequenced questions guide the student's thinking through an argument that allows to reach certain conclusions, convergent thinking, thus a more dynamic and participatory exposure is performed.

The practical work-integrated projects of the units of the various agenda items must contain not only implementation but also activities of analysis and discussion, tending to integrate the theoretical and practical, so proceed from reflection to action and then the improvement of the action. The teacher acts as a facilitator of learning ", leading to questions, presenting situations, pointing out mistakes and avoiding address what the student can solve by themselves. The situations should be simpler in the first stage gradually increasing its complexity [10].

# 4 PROJECT PLANNING

Project based learning has proved to be a suitable method to demonstrate the need of mathematics in professional engineering. Students are confronted, complementary to their regular courses, with problems that are of a multidisciplinary nature and demand a certain degree of mathematical proficiency [2].

The curricula of the Mechanical Engineering programs at our university include Advance Calculus at the third year of the engineering career; the authors' experience is that students increased their interest and their appreciation for the contents if they are involved by learning in an applied way.

The project proposal was a selection of contents related with Mechanics Fluids, by employing a platform of multiphysics simulation where Mathematics and varied Physics systems are involved to model the system behavior.

The simulation of different fluid behaviours is a technique widely used in the majority of the industries, being the computational fluid dynamics (CFD) one of the techniques that uses numerical methods and algorithms to replace the partial differential equation systems into algebraic equation systems to be resolved by the aid of computers.

CFD techniques provide qualitative and quantitative information about fluid flow prediction by means of resolving fundamental equations, allow predicting or simulating behaviours in a virtual laboratory.

Using CFD is possible to build a computational model that represents a system to study, specifying the physical and chemical fluid conditions of the virtual prototype and the software will deliver a prediction of the fluid dynamic, therefore is a design and analysis technique implemented by a computer. The main advantages are:

- Predicts the fluid properties with great detail in the studied domain.
- Helps design and prototype with quick solutions avoiding costly experiments.
- Process visualization and animation can be obtained in terms of the fluid variables [9].

The proposal presented is intended to explore potential theory examples and discuss some fluids that can be approximated using Computational Fluid Mechanic. When the potential flow presents complicated geometries or unusual current conditions, the conformal transformation based in complex variables cease being useful to generate forms of bodies. In this case the numerical analysis technique constitutes a more appropriate approach.

The finite difference method for the potential flow has the aim to approximate partial derivatives listed in a physical equation by "the difference" between the values of the solution in a number of modes spaced by some certain finite distance. The original equation in partial derivatives is replaced by a series of algebraic equations for the nodal values.

The system being studied is the two-dimensional flow of an incompressible fluid, non-viscous corresponding to an irrotational field that moves in a steady state for different values of the potential velocity complex. Using complex function models, an analysis of the behaviour of the fluid is done to conceptualise the subject and then is contrasted with the introduction of CFD technique:

## 4.1 1st Session: Theoretical research of the topic

Groups of four students were formed with the suggested bibliographic material ([4], [6], [7], [8], [12]) and with an accompanying teacher, we came to understand and obtain a tutorial guide of theoretical concepts used in the proposed work, upon request, after hearing and reviewing the conclusions reached by each group, a debate about the subject took place proposing the following framework:

(i) A complex variable function $f(z) = p(x,y) + jq(x,y)$ univocal some region $R$ of the plane $z$, is analytic in the region $R$ if a derivative $f'(z)$ exists at every point $z$ in that region $R$.

(ii) A necessary condition for that $f(z)$ is analytical in a region $R$, is that in $R$, $p$ and $q$ satisfy the Cauchy-Riemann equations.

$$\frac{\partial p}{\partial x} = \frac{\partial q}{\partial y} \quad ; \quad \frac{\partial p}{\partial y} = -\frac{\partial q}{\partial x} \qquad (1)$$

If these partial derivatives are continuous in $R$, then the above equations are sufficient conditions to be $f(z)$ analytical in $R$. The functions that satisfy the above conditions are said conjugate and such functions satisfy the orthogonality property means that the type curves $p(x,y) = k_1$, and $q(x,y) = k_2$, are orthogonal.

(iii) In addition if the second partial derivatives $p$ and $q$ are continuous in $R$, and Laplace equation is meet for $p$ and $q$:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0 \quad ; \quad \frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial y^2} = 0 \qquad (2)$$

(iv) The derivative of the function $f$ can be calculated by the following expression:

$$f'(z) = \frac{\partial p}{\partial x} + j\frac{\partial q}{\partial x} = -j\frac{\partial p}{\partial y} + \frac{\partial q}{\partial y} \qquad (3)$$

## 4.2 2nd Session: Modeling the system

The velocity complex potential $F$ is an analytical function of complex variable composed of the ordered pair whose real part is the velocity potential and imaginary part is the current function $\varphi$:

$$F(z) = \phi(x,y) + j\varphi(x,y) \qquad (4)$$

The fluid flow velocity is $v = p + jq$ , is the gradient of the potential velocity :

$$\nabla\phi = v \quad (5)$$

From equations (4) and (5) we could express that the fluid velocity in the Eq. (6):

$$v = \phi_x(x,y) + j\phi_y(x,y) \quad (6)$$

Differencing the complex potential from Eq. (4) was obtained Eq. (7):

$$F'(z) = \phi_x(x,y) + j\varphi_x(x,y) \quad (7)$$

As $F$ is an analytic function meets the conditions expressed by the Cauchy-Riemann equations it can be concluded that:

$$\frac{\partial\phi}{\partial x} = \frac{\partial\varphi}{\partial y} \quad ; \quad \frac{\partial\phi}{\partial y} = -\frac{\partial\varphi}{\partial x} \quad (8)$$

In this case the equipotential curves are those such that: $\phi(x,y) = k_1$, and are orthogonal to the streamline which are equation curves $\varphi(x,y) = k_2$ , (with $k_1$ y $k_2$ constants).

With the model accomplished and by Eq. 7 and Eq. 8 it could be concluded that the fluid velocity is a conjugated complex, derivate from the $F$ function.

$$F'(z) = \phi_x(x,y) - j\phi_y(x,y) \quad (9)$$

It expresses Eq. (9) also by means of Eq. (10):

$$v = \overline{F'(z)} \quad (10)$$

In order to calculate the magnitude of the velocity $|v|$ was used Eq.(11).

$$|v| = |F'(z)| \quad (11)$$

## 4.3 3rd Session: Analysis and discussion of different cases

### *4.3.1* $F(z) = V_o\left(z + \frac{a^2}{z}\right)$

We propose to study the movement of a known fluid, for different expressions of the complex potential, considering in every case the constant $a > 0$. Where $V_o$ is a positive constant.

We guide the students work analysis with the following instructions ([1], [3], [11]):

a) Obtain the equations for the streamlines and equipotential lines.
b) Make a graphical representation of the previous paths and interpret them physically
c) From the graphs obtained in the previous section, discuss how the fluid regime is.
d) Analyse how the velocity profile will be at different points of his path.
e) Individualize the stationary points.

From the proposed analysis and by the graph of Fig. 1 the following findings were established:

(i) The axis parallel curves indicate paths that fluid particles follow. If $\varphi = 0$ the path which moves in the $x$ axes, or in the contour of circle of radius $a$ .
(ii) Equipotential lines are marked with dotted lines and are orthogonal to the streamlines.
(iii) The circumference of radio $a$ represents a path, and since there can't be a flow through a path, it can be considered as a circular obstacle of radius $a$ placed in the fluid path.
(iv) The complex velocity of the fluid has a variable value near the obstacle and its module corresponds to $|v| = V_o\sqrt{1 + \frac{a^4 - 2a^2\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2}}$

(v) If we move away from the obstacle the velocity takes the value $v = V_o$, i.e. the fluid is running on the positive $x$ axis direction with constant velocity $V_o$.

(vi) The stationary points of the system are those where the velocity is zero and are given by the values of $z = a$ and $z = -a$.

### *4.3.2* $F(z) = az$

From the analysis of Fig. 2 we been able to visualize the behaviour of the fluid in this case, where the power lines are horizontal lines and the corresponding equipotential curves are vertical lines, indicating that the fluid flow is uniform and its direction is right, this is also interpreted as a uniform flow in the upper half plane bounded by the $x$ axis, which is a streamline, or a uniform flow between two parallel lines $y = k_1$ and $y = k_2$, with its velocity $v = a$, and its module of the same value.

### *4.3.3* $F(z) = az^2 \ (x > 0, y > 0)$

From observing Fig. 3, we see that the fluid is forced to rotate towards a corner located at the origin. In this case, the streamlines are branches of rectangular hyperbolae responsive to the equality $2axy = k$, so that the velocity module is directly proportional to its distance from the origin, being its expression $|v| = 2a|z|$ . It was concluded that the value of the stream function at a point is interpreted as the flow rate through a line segment that joins the origin with that point.

### *4.3.4* $F(z) = ja \ln z$

Streamlines analysed in Fig. 4 are circles having a common centre which corresponds to the origin of the complex plane ($z = 0$) while the equipotential lines are given by the lines of equations $y = kx$, which also pass through the origin. Thus the complex potential describes the flow of a fluid that is circling around is called "vortex" and this type of flow is called "vortex flow". In this case the direction flow matches clockwise direction and the magnitude of the velocity is: $|v| = \dfrac{a}{\sqrt{x^2 + y^2}}$

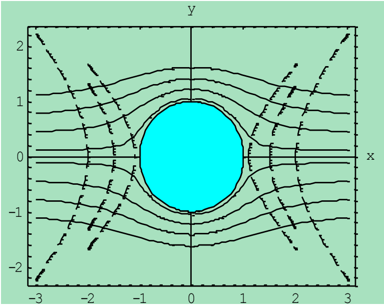
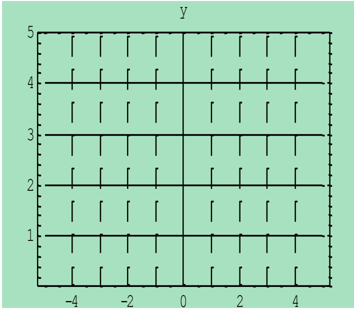
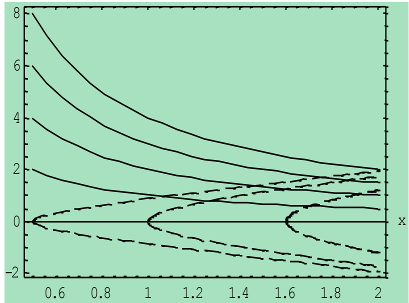
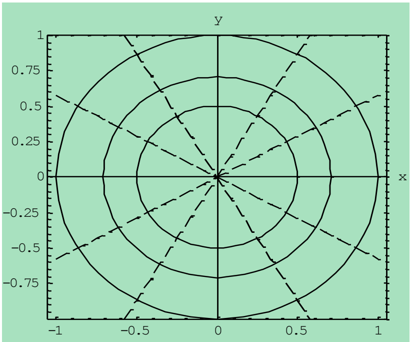
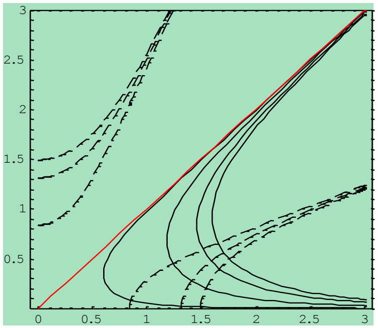
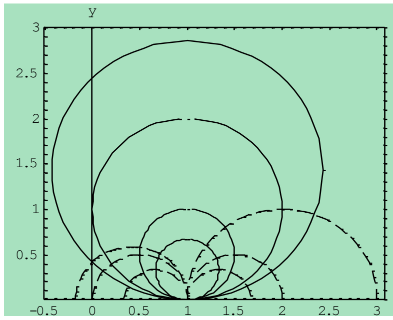
### *4.3.5* $F(z) = az^4$

As shown in Fig. 5, we study the region in the first quadrant, in this case we observed that the streamlines for this region are bounded by the $x$ axis and the line $y = x$. The direction flow is downward and the value of the module of the velocity is $|v| = 4a\left|z^3\right|$.

### *4.3.6* $F(z) = a\dfrac{1+z}{1-z}$

In this case Fig. 6 shows that the streamlines are concentric circles around a point and that from that point equipotential curves are born. It appeared that the velocity module corresponding to the formula is: $|v| = \dfrac{2}{x^2 + y^2 - 2x + 1}$

Table I: Streamlines and equipotential curves.

| | | |
|---|---|---|
|  |  |  |
| Fig1 Case 4.3.1 | Fig. 2 Case 4.3.2 | Fig. 3 Case 4.3.3 |
|  |  |  |
| Fig. 4 Case 4.3.4 | Fig.5 Case 4.3.5 | Fig. 6 Case 4.3.6 |

## 5 THE IMPACT OF APPLYING CFD TECHNOLOGY

Taking the proposal of work done in section 4.3.1 the simulation is performed with the Creeping Flow module of COMSOL Multiphysics. The working environment of the program allows adding to the proposed model, new physical parameters besides proposing, evaluating and exchanging boundary conditions. The potentiality that the program has, contributed to work with educational proposals that approximate to reality aside from the ideal conditions.

### 5.1 Simulated domain and physical conditions

By 6 cm diameter UNS C10100 bronze pipe, water flows in a laminar regime with a Reynolds number less than 2500 and a temperature of 20 ° C. In this pipe there is a blockage of a sphere of alloy steel 1006 (UNS G10060) of 0.3 cm radius. If the pressure ranges from 3 to 2 Pa in a section of 10 cm length, and it is considered that the fluid has viscosity on the outer walls and roughness in the sphere, it is requested to study the variations in the pressure and velocity caused by such obstruction.

Using conformal mapping techniques based on complex variable techniques there are no friction losses and the fluid is considered in ideal conditions, with COMSOL platform is not possible to consider the value of zero viscosity.

### 5.2 Prototype Geometry and Meshing

As shown in Fig. 7, tools that the program has to perform rendering of geometry working are used. However, we must note that the software allows importing models with more complex designs from design programs in solid such as Solid Works, Space Claim and Inventor. This interaction between programs is very useful for the design of complex geometries.

The procedure for performing the profile graph of the shape of the pipe is: define the dimensions of the rectangle, the size and position of the sphere, and use the intersection tool to complete the geometry.

Choosing a correct meshing is of utmost importance to verify the accuracy of the results. To make the mesh, it is taken into account the shape and the maximum and minimum measurements of geometry to study. Prior to making the modeling, we proceed to the meshing of the geometry used, to visualized in Fig. 8, the standard predefined triangular free mesh made automatically has been opted.

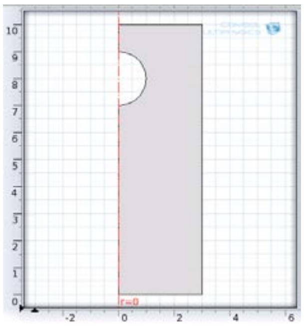


Fig.7. Prototype Geometry

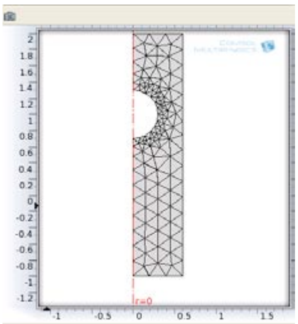


Fig. 8 Meshing

## 5.3 Results

Fig. 9, Fig. 10 and Fig. 11 shown the formation from the tube walls of a profile with increasing velocity called boundary layer. In the central part, where the sphere is positioned, zero velocity are noticed at the front and rear of it, indicating the absence of fluid flow in that area and as a result of this, significant pressure fluctuations.

Fig. 12, Fig. 13 and Fig. 14 show that at the side of the sphere there is a considerable pressure drop which results in an increase in velocity due to the decreased cross-sectional area. This situation is similar to what occurs in other measuring instruments such as flow rates and Venturi tube, nozzle and orifice plates.

In the front of the sphere the absence of velocity causes the maximum pressure in this area, because of this it can be understood the basis of certain measuring instruments that use stagnation of fluids, such as the Pitot tube used to measure total pressure and the Prandtl tube used for measuring dynamic pressure and velocity.

At the rear of the sphere there is a low pressure zone, which can cause the detachment of the boundary layer. This may happen or not depend on each case from the fluid conditions and roughness of the sphere.

Even though graphics of pressure and velocity have been made over time, the program also allows evaluating specific points or features of the model and finding the maximum and minimum values of both velocity and pressure.

As for the model used, as long as the student acquires greater expertise a higher degree of problem complexity could be achieved, varying border conditions or using this model as a starting point for studies in nozzles, diffusers or in other engineering applications.

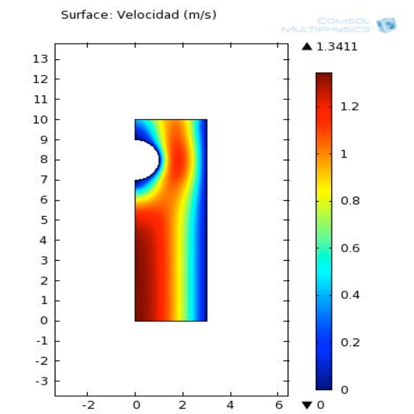



Fig. 9 Velocity level surfaces

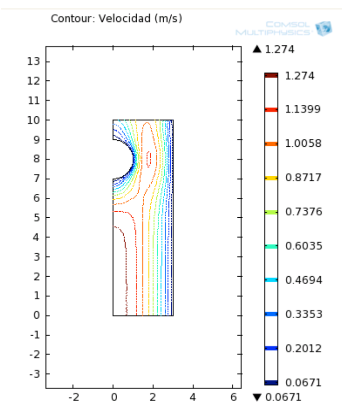



Fig. 10: Velocity level curves

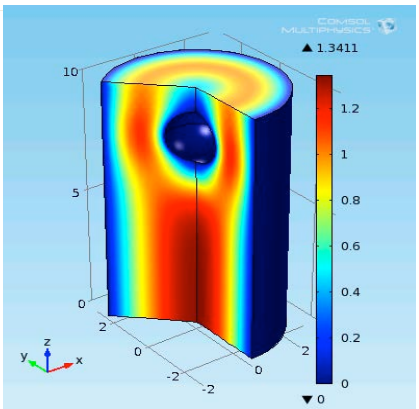
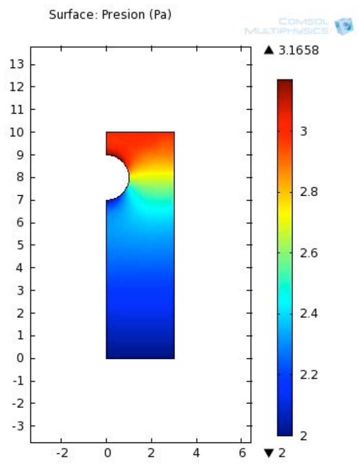
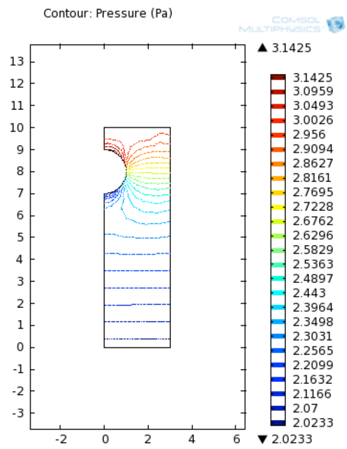



Fig. 11: 3D velocity level surfaces



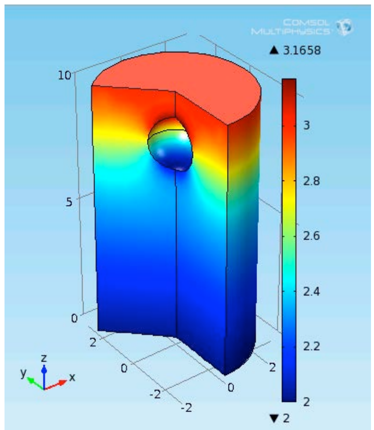
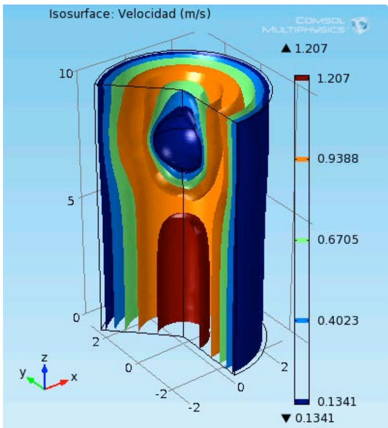




Fig. 12: Level pressure surface  Fig. 13: Pressure level curves  Fig. 14: Level pressure surfaces

In Fig. 15 and Fig. 16 depicts graphs of the level surfaces corresponding to the velocity and pressure of each level.



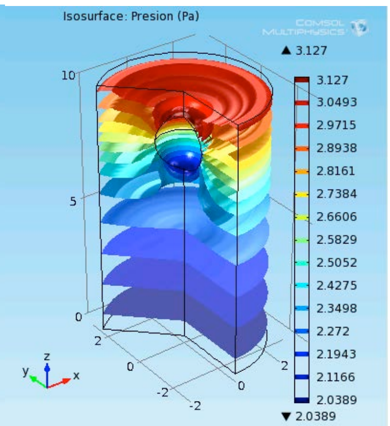




Fig. 15 Velocity level surfaces  Fig. 16 Pressure level surfaces

Besides the above, the narrowing of sections, such as the one caused by the sphere, are the basis of some instruments operation such as ejectors, used to mix fluids and accelerate fluids.

The observed variations also allow to understand why we recommend placing the measuring instruments to 5 times the minimum diameter of the pipe, from any disturbance so as to achieve results biased by them.

## 6 CONCLUSIONS

The use of analytical functions of complex variables, the study of idealized models and support of computer resources used appropriately lead to a contextualized analysis which achieve the goal of understanding the possibilities of integration between computational mathematics and fluid mechanics.

These laboratory experiences are a link between basic science and applied technologies that equip students an autonomous character to work with problem situations similar to their future professional work.

Through numerical simulation and basic knowledge of fluid flow simple problems are introduced using an environment such the one offered by COMSOL platform to assess the importance of the results obtained in an analytical and symbolic form, and the possibilities for future use of the virtual learning laboratory, in applied technologies studies, to support disciplinary integration activities.

This methodology allows deepening and integrating the basic concepts, as well as arousing student interest in the incorporation of new topics, to be able to show from the early years of their careers and present practical applications of recent development work which may lead to a constant search for new knowledge.

The intense activity has also allowed the incorporation of young students for training in both teaching and research, who work on activities that are carried out in the laboratory and approaching concerns and addressing engineering problems in the subjects of upper cycle.

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